Q.P. Code : 60866

Second Semester M.Sc. Degree Examination, July 2019

(CBCS Scheme)

Mathematics

Paper M 207 SC – ELEMENTARY NUMBER THEORY

Time : 3 Hours] *[Max. Marks : 70*

Instructions :

1) *Answer any* **FIVE** *full questions.*
2) *All questions carry equal marks.*

1. (a) State and prove Division Algorithm. (7)

   (b) Determine all solutions in the set of integers of the disphantine equation $221x+35y=11$. (7)

2. (a) If the square root of a positive integer '*m*' is rational, then show that '*m*' is a perfect square. (7)

   (b) (i) Prove that there are infinitely many primes of the form $4q+3$.

   (ii) An integer $n>1$ is composite iff it is divisible by some prime $p\leq\sqrt{n}$. (7)

3. (a) Prove that relation 'Congruence modulo *n*' is an equivalence relation on the set of integers. (7)

   (b) Find all integers '*n*' for which $n^{13}\equiv n\pmod{1365}$. (7)

4. (a) State and prove

   (i) Euler-Fermat theorem

   (ii) Fermat little theorem (7)

   (b) Solve the simultaneous congruences :

   (i) $x\equiv 5\pmod{11}$

   $x\equiv 14\pmod{29}$

   $x\equiv 15\pmod{31}$

   (ii) $x^2+2x+2\equiv 0\pmod{5}$

   $7x\equiv 3\pmod{11}$ (7)

5. (a) Let 'p' be a odd prime, then prove that $(mn/p) = (m/p)\ (n/p)$ for all m, n integers. (7)

(b) For every odd prime p, then prove that (7)

$$(2/p) = (-1)^{\frac{p^2-1}{8}} = \begin{cases} 1 & \text{if } p \equiv \pm 1 \pmod 8 \\ -1 & \text{if } p \equiv \pm 3 \pmod 8 \end{cases}$$

6. (a) If P and Q are odd positive integers, then prove that

(i) $(m/P)(n/P) = (mn/P)$

(ii) $(n/P)(n/Q) = (n/PQ)$

(iii) $(m/P) = (n/P)$ if $m \equiv n \pmod p$ (7)

(b) Let 'P' be an odd prime and let $\gcd(a, p) = 1$. If n denotes the number of integers in the set $S = \left\{a, 2a, 3a, \ldots\ldots\ldots, \left(\frac{p-1}{2}\right)a\right\}$, whose remainders upon division by P exceeds $P/2$, then prove that $(a/p) = (-1)^n$. (7)

7. (a) Prove that an odd prime 'p' is expressible as sum of 2-squares iff $P \equiv 1 \pmod 4$. (7)

(b) (i) If m and n are both sum of 2-squares then prove that their product mn is also a sum of two squares.

(ii) Prove that no prime p of the form $4K + 3$ is a sum of 2-squares. (7)

8. (a) If P is an odd prime, then the congruence $x^2 + y^2 + 1 \equiv 0 \pmod p$ has a solution x_0, y_0 where $0 \le x_0 \le \frac{(p-1)}{2}$ and $0 \le y_0 \le \frac{(p-1)}{2}$. (7)

(b) Show that the Diophantine equation $x^4 + y^4 = z^2$ has no solution in positive integers x, y, z. (7)
